On A-computable Families: Numberings, Rogers and Degtev Semilattices

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This talk is concerned with the computational properties of families of subsets of $\mathbb{N}$. Recall that any surjective mapping from $\mathbb{N}$ onto a countable family $\mathcal{F} \subseteq 2^{\mathbb{N}}$ is called a *numbering* of $\mathcal{F}$. Following [1, 2], we say that, for a given set A, a numbering ν is *A-computable* if there is a computable function f such that $\nu(x) = W^A_{f(x)}$ for each $x \in \mathbb{N}$. Families with A-computable numberings are also called *A-computable*. A numbering ν is said to be *reducible* to a numbering μ ($\nu \leqslant \mu$) if $\nu = \mu \circ f$ for some computable function f. Two numberings ν and μ are said to be *equivalent* ($\nu \equiv \mu$) if they are reducible to each other. *The Rogers semilattice* of an A-computable family is the quotient structure of its A-computable numberings with respect to the equivalence of numberings.

The Rogers semilattice of a computable family can be viewed as an algebraic reflection of its effective topological properties. For example, the Rogers semilattice of any computable effectively discrete family is one-element, but the Rogers semilattice of any finite family with two sets comparable under inclusion is infinite. In the first part of the talk, we will present results on Rogers semilattices of A-computable families for any non-computable oracle A. Namely, we will consider questions about their possible cardinalities, their latticeness, their distributivity, the existence of their greatest and minimal elements, etc.

In the second part of the talk we will consider semilattices of all computable families and all A-computable families, where A is an arbitrary oracle. These semilattices were introduced and first studied by A.N. Degtev [3]. We will consider questions about the definability of their Fréchet ideals, about the existence of their nontrivial definable singletons, and formulate some related questions.

References

[1] S.S. Goncharov, A. Sorbi, Generalized computable numerations and non-trivial Rogers semilattices, Algebra and Logic, **36**:6 (1997), 359–369.

[2] S.A. Badaev, S.S. Goncharov, Generalized computable universal numberings, Algebra and Logic, **53**:5 (2014), 355–364.

[3] A.N. Degtev, The semilattice of computable families of recursively enumerable sets, Mathematical Notes of the Academy of Sciences of the USSR, **50** (1991), 1027–1030.